அரசினர் ஆடவர் கலைக்கல்லூரி (தன்னாட்சி), நந்தனம், சென்னை – 35.

CODE: **196108**
NOVEMBER 2020

TIME: 3 Hrs
MAX. MARKS: 50

PART A *(10 x 2=20)*

*Answer any **TEN** questions.*

1. Define Jordan arc.
2. Define conformal mapping.
3. Define the index of the point a with respected to the closed curve γ.
4. Show that $z = \infty$ is an essential singularity of e^z.
5. State Rouche's theorem.
6. State Cauchy residue theorem.
7. Define Potential function.
8. Show that$a\ logr + b$, where $a\ and\ b$ are constants, is harmonic.
9. State Weierstrass theorem.
10. . State Poisson Integral of U.
11. .State the Reflection principle.
12. . Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$

PART B *(2x 5=10)*

*Answer any **TWO** questions.*

13. Prove that the line integral$\int_{\gamma}\ pdx + qdy$ define in Ω,depends only on the end point of γ if and only if there exists a function , $U(x, y) in\ \Omega$ with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.
14. State and prove Taylor series for an analytic function.
15. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2
16. Evaluate $\int_0^{\pi} \frac{d\theta}{a+cos\theta}$,$a > 1$.
17. (i) Prove that the function $P_U(z)$ is harmonic for $|z| < 1$. and $\lim_{z \to e^{i\theta_0}} p_U(z) = U(\theta_0)$ Provided that U is continuous at θ_0
18. State and prove Hurwitz theorem.
19. State and prove the Maximum Principle.
20. Evaluate:$\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.

PART C *(2 x 10=20)*

*Answer any **TWO** questions.*

21. State and prove Cauchy theorem in a disk.
22. Suppose that $\varphi(\zeta)$ is continuous on the arc γ. Then the function $F_n(z) = \int_{\gamma} \frac{\varphi(\zeta)d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ and its derivative is $F'_n(z) = nF_{n+1}(z)$.
23. State and prove Argument of Principle.
24. State and prove Poisson's formula for a function harmonic in an open disk
25. State and prove Laurent's series.

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