CODE: **196103** TIME: 3Hrs

NOVEMBER 2020 MAX. MARKS : 50

PART A *(10 x 2=20)*

Answer any ***TEN*** *questions*

1. Find all the solutions of $y'' - 4y' + 5y = 0$.
2. Show that the functions $\varphi_1(x) = x, \varphi_2(x) = xe^x$ are linearly independent on $-\infty < x < \infty$.
3. Find all solutions of $y''' - 8y = 0$
4. Using the Annihilator method find a particular solution of the equation $y''' = x^2$.
5. Show that $\varphi_1(x) = x^2$ is one solution of the equation $y'' - \frac{2y}{x^2} = 0$
6. Show that the Legendre polynomial $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$.
7. Find all solutions of the equation $x^2y'' + xy' + 4y = 1$ for $|x| > 0$.
8. Write the Bessel function of zero order of the second kind.
9. Solve : $y' = \frac{e^{x-y}}{1+e^x}$.
10. State Lipschitz condition.
11. Define linearly independent.
12. Let φ_1, φ_2 be two solutions of $L(y) = 0$ on an interval I,and let x_0 be any point in I.Prove that φ_1, φ_2 are linearly independent on I if $W(\varphi_1, \varphi_2)(x_0) \neq 0$.

PART B *(2 x5=10)*

Answer any ***TWO*** *questions*

13. State and prove Existence theorem for initial value problem involving second order homogeneous ordinary differential equation with constant coefficients.
14. Using the Annihilator method find a particular solution of the equation $y'' + 4y = sin2x$.
15. Let $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n$ be n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \dots + a_1(x)y = 0$ on an interval I.Prove that if $\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n$ are Linearly independent then $W(\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n)(x_0) \neq 0$.
16. Prove that $j_{1/2}(x) = \sqrt{\frac{2}{\pi x}}(cox)$
17. Solve : $(2ye^{2x} + 2xcosy)dx + (e^{2x} - x^2siny)dy = 0$.
18. Find the solution of the initial value problem $y'' - 2y' - 3y = 0, y(0) = 0, y'(0) = 1$.
19. State and prove Uniqueness theorem of the initial value problem for n –th order linear homogeneous equation with variable coefficients.
20. Find the singular point of the equation $(1 - x)^2y'' - 2xy' + 2y = 0$ and determine whether they are regular singular points or not?

PART C (2x10=20)

*Answer any **TWO** questions*

21. Find all solutions of $y'' - 2y' = e^x sinx$

22. Let ϕ be any solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval *I* containing a point x_0.Then prove that for all *x* in *I*, $\|\phi(x_0)\|e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\|e^{-k|x-x_0|}$

where $\|\phi(x)\| = ([\ |\phi(x)|^2 + \ |\phi'(x)|^2)^{\frac{1}{2}}\ ])$, $k = 1 + |a_1| + |a_2|$.

23. Show that $\int_{-1}^{1} P_n{}^2(x)dx = \frac{2}{2n+1}$.

24. Find a solution ϕ of the form $\phi(x) = x^r \sum_{k=0}^{\infty} C_k x^k, (x > 0)$ for the equation $x^2y'' + \frac{3}{2}xy' + xy = 0$

25. Prove that the necessary and sufficient condition for an equation $M(x,y) + N(x,y)y' = 0$ to be exact.