அரசினர் ஆடவர் கலைக்கல்லூரி (தன்னாட்சி), நந்தனம், சென்னை – 35.

CODE: **196102** TIME: 3Hrs

NOVEMBER 2020 MAX. MARKS : 50

PART A *(10 x 2=20)*

*Answer any **TEN** questions*

1. Define partition on interval [a, b].
2. Define total variation.
3. Define the Riemann-Stieltjes integral.
4. What is Riemann's condition?
5. State second fundamental theorem of integral calculus.
6. State first mean value theorem for Riemann-Stieltjes integrals.
7. Define uniform convergence for sequence of functions.
8. State Cauchy condition for uniform convergence of the series.
9. State weierstrass M test for convergence.
10. Define boundedly convergent..
11. Define power series..
12. Write Taylor's series..

PART B *(2 x5=10)*

*Answer any **TWO** questions*

13. If f is monotonic on [a, b] then prove that the set of discontinuities of f is countable.
14. If $\alpha \uparrow$ on [a,b] then prove that $\underline{I}(f,\alpha) \leq \overline{I}(f,\alpha)$.
15. State and prove second mean value theorem for Riemann-Stieltjes integrals
16. If $f_n \to f$ uniformly on S. and each f_n is continuous at a point C of S, then prove that the limit function f is also continuous at C.
17. State and prove Bernstein theorem.
18. If f is continuous on [a, b]. Then f is bounded variation on [a, b], iff f can be expressed as the difference of two increasing functions.
19. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a, b], then $c_1 f + c_2 g \in R(\alpha)$ on [a, b] and we have

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$$\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b f\, dx + c_2 \int_a^b g\, dx.$$

20. State and prove Bonnet's theorem

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PART C *(2x10=20)*

*Answer any **TWO** questions*

21. Let f be of bounded variation on [a, b] and assume that $c \in (a,b)$. Then prove that f is of bounded variation on [a, c] and on [c, b] and, $v_f(a,b) = v_f(a,c) + v_f(c,b)$.

22. If $\alpha \uparrow$ on [a,b]. Then the following three statements are equivalent

(i). $f \in R(\alpha)$ on [a, b]

(ii). f satisfies Riemann′s condition with respect to α on [a, b].

(iii). $\underline{I}(f, \alpha) = \bar{I}(f, \alpha)$.

23. State and prove second mean value theorem of Riemann integral.

24.State and prove the Cauchy criterion for uniform convergence for function of sequence.

25.If f has a continuous derivative of under n+1 in some open interval I containing C, and define E_n(x) for x in I, then prove,

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k + E_n(x). \text{ Where, } E_n(x) = \frac{1}{n!}\int_c^x (x-t)^n f^{(n+1)}(t)dt.$$
