அரசினர் ஆடவர் கலைக்கல்லூரி (தன்னாட்சி), நந்தனம், சென்னை – 35.

CODE: **196101** TIME: 3Hrs

NOVEMBER 2020 MAX. MARKS : 50

PART A *(10 x 2=20)*

*Answer any **TEN** questions*

1. Define conjugate of *a* in *G*.
2. Show that $a \in Z(G)$ if and only if *N(a)=G*.
3. Define cyclic *R*-module
4. Define internal direct product of normal subgroup of *G*.
5. Define kernel of a homomorphism.
6. Define principle ideal ring.
7. Define vector space.
8. Define algebraic of degree *n* over *F*.
9. State Remainder theorem.
10. For any $f(x), g(x) \in F[x]$, prove that $\left(f(x)+g(x)\right)' = f'(x)+g'(x)$.
11. Define finitely generated *R*-Module.
12. Define Greatest common divisor of a ring.

PART B *(2 x5=10)*

*Answer any **TWO** questions*

13. State and prove second part of sylow's theorem
14. Suppose that *G* is the internal direct product of $N_1, N_2, \dots, N_n$ .Then for $i \neq j, N_i \cap N_j = (e)$,and if $a \in N_i, b \in N_j$,then prove that $ab = ba$.
15. Prove that a Euclidean ring possesses a unit element.
16. If *V* is a vector space over *F* then prove that
    (i) $\alpha 0 = 0 \ for \ \alpha \in F$.
    (ii) $0v = 0 \ for \ v \in V$.
    (iii) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F, v \in V$.
    (iv) If $v \neq 0$,then $\alpha v = 0$ implies that $\alpha = 0$.
17. Prove that if $F(a)$ is a finite extension of *F* then the element $a \in K$ is algebraic over *F*.
18. Prove that $N(a)$ is a subgroup of *G*.
19. Show that the external direct product of two groups *A* and *B* is also a group.
20. State and prove division algorithm.

P.T.O

PART C *(2x10=20)*

*Answer any **TWO** questions*

21. State and prove Cauchy theorem.
22. If R is an Euclidean ring then show that any finitely generated R-module M is the direct sum of a finite number of cyclic submodules.
23. State and prove Unique factorization theorem.
24. Prove that the number "e" is transcendental.
25. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.
