அரசினர் ஆடவர் கலைக்கல்லூரி (தன்னாட்சி), நந்தனம், சென்னை – **35.**

CODE: **196109** TIME: 3 Hrs

NOVEMBER 2020 MAX. MARKS: 50

PART A *(10 x 2=20)*

*Answer any **TEN** questions.*

1. Define Normed linear space.
2. Define bounded linear transformation.
3. Define Hilbert space with an example.
4. State open mapping theorem.
5. Prove that $(T_1T_2)^* = T_2{}^*T_1{}^*$
6. If N is normal operator on H then prove that $\|N^2\| = \|N\|^2$.
7. Define Banach Algebra.
8. Prove that if r is an element of R then 1-r is left regular.
9. Define Involution in Banach algebra.
10. .If x is a normal element in a B^* -algebra then prove $\|x^2\| = \|x\|^2$
11. . Prove that every maximal left ideal in A is closed.
12. . State closed graph theorem.

PART B *(2 x 5=10)*

*Answer any **TWO** questions.*

13. Let M be a closed linear subspace of a Normed linear space N. If the norm of a coset x+M in the quotient space N/M is $\|x+M\|$= inf $\{\|x+m\|: m \in M\}$ then, show that N/M is a Normed linear space.
14. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm
15. Prove that If T is an operator on H for which $(Tx,x) = 0$ for all x *then* $T = 0$.
16. Prove that $\sigma(x)$ is non-empty.
17. Prove that if f_1 and f_2 are multiplicative functional on A with same null space then $f_1 = f_2$.
18. Show that the mapping $x \to x^{-1}$ *of* G *into* G is continuous and is therefore a homeomorphism of G *onto itself*.
19. State and prove Schwarz inequality.
20. If P is a projection on H with range M and null space N,then M $\perp N \Leftrightarrow P$ is self adjoint, and in this case $N = M^{\perp}$.

PART C *(2 x 10=20)*

*Answer any **TWO** questions.*

21. State and Prove the Hahn –Banach theorem.
22. State and prove the Uniform boundedness theorem.
23. Prove that ,If H is a Hilbert space. and let f be an arbitrary functional in H^*.Then there exists a unique vector y in H such that $f(x) = (x,y)$ *for every* x *in* H.
24. Show that $r(x) = lim\|x^n\|^{\frac{1}{n}}$.
25. State and prove Gelfand –Neumark representation theorem.

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